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A FORWARD AND INVERSE TRANSFORMATION PROGRAM FOR THE "ATLAS OF LITHOLOGICAL–PALEOGEOGRAPHICAL MAPS OF THE WORLD"

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Abstract—A computer program is presented that performs forward and inverse transformations of the polyconic map projection used for the world maps in the "Atlas of Lithological–Paleogeographical Maps of the World". The program is intended as a tool for setting up data sets in digital form from this atlas that can be used for paleoclimate modeling and sedimentary mass-balance studies. The accuracy of the forward transformation is $\leqslant 0.2$ mm for most parts of the maps. The program runs under DOS on a Personal Computer (PC) and can be used interactively or in batch mode. Digitized map data are output as spherical coordinates (longitude–latitude) either as plain ASCII file or in the format of the ATLAS program.

Key Words: Polyconic map projection, Forward/inverse transformation, Paleoclimate proxy data.

INTRODUCTION

The "Atlas of Lithological–Paleogeographical Maps of the World" for the Mesozoic and Cenozoic (Ronov, Khain, and Balukhovsky, 1989) is an important resource for paleoclimate research because it shows global topography, shorelines, distribution of sediments, and occurrence of climate-sensitive deposits such as evaporites, coals, laterites, bauxites, redbeds, and reefs. The atlas has been used as a source of data for both boundary conditions and validation of paleoclimate models (Wilson and others, 1994; Hay and others, 1994). It also has been used in studies of sedimentary mass-balance, for example, the determination of areas of different types of rock exposed to erosion (Bluth and Kump, 1991). However, the forward and inverse transformations for the map projection used for the world maps in this atlas has not been published. Hence, efficient digitization of these maps for the purpose of establishing datasets of paleoclimate proxy data in digital form was not possible. Note that forward transformation translates spherical coordinates (longitude–latitude) into the cartesian (rectangular) coordinates of a given map projection, whereas an inverse transformation translates cartesian coordinates into spherical coordinates (e.g. Snyder, 1987, p. 29–32).

In the following we will present approximate equations for the forward transformation and briefly describe an algorithm for the inverse transformation. The numerical accuracy of our approach is sufficient for setting up digitized versions of the mentioned atlas for the purpose of paleoclimatic modeling. However, we would emphasize that our approach might not be sufficient to meet the requirements on accuracy posed by cartographers.

FORWARD TRANSFORMATION

The world maps in the atlas are based on a polyconic projection developed at the Central Scientific Research Institute of Geodesy, Air Survey, and Cartography in Moscow (TsNIIGAiK). The projection being used belongs to a class of modified polyconic projections that evolved between 1939 and 1954 (see Maling, 1960 for a review).

The resulting world maps are centered on the equator and 40°E (the central meridian), and are shown on a map scale of 1:48,000,000. Some features of this map projection are the following: (1) the equator extends from the left-hand margin of the map at 140°W, to the east through 180°E and has its right-hand boundary at 140°W (or 220°E). This

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results in Alaska being shown twice on the maps in the upper left- and right-hand corners. (2) All lines of latitude and longitude are curved except for the equator and central meridian. (3) Parallels of latitude are nonconcentric circular arcs. (4) Meridians are spaced equally along a given parallel of latitude. (5) The scale is true along the parallels at 45°N and 45°S.

Originally, the forward transformation was achieved by the following procedure (cf. Maling, 1960): Given a set of three polynomial equations, it is possible to determine the cartesian coordinates of the intersections of parallels of latitude with the central meridian, and for the location (again in cartesian coordinates) of the same parallel at a distance of 180° from the central meridian. Using the facts that the parallels of latitude are circular and that the spacing of meridians along any parallel of latitude is even, it would be possible to derive equations for the forward transformation for any given point. Although this approach would yield the most accurate results, it cannot be followed in practice. This is because the polynomial coefficients that were used in the previously mentioned equations have not been published in the atlas. The reason is that the preparation of the maps was done by using already existing tables of spherical vs. cartesian coordinates instead of an analytical derivation for the forward transformation.

In order to establish approximate equations for the forward transformation, we measured (using a digitizing tablet) the cartesian coordinates of all ($N = 171$) intersections of the longitude–latitude grid on three different maps. Averaging the three measurements resulted in a set of cartesian coordinates for given spherical coordinates. Instead of making triplicate measurements using a single map, we used three different maps to account for possible differences between the maps due to printing and variable expansion coefficients of the map paper (cf. Snyder, 1987, p. 3). The standard error of the triplicate measurements is in the order of 0.1 mm.

Having set up a table of cartesian coordinates and the corresponding spherical coordinates, the aim then is to determine empirical relationships between these variables of the form $x = f_x(\lambda, \phi)$ and $y = f_y(\lambda, \phi)$. Here x denotes the eastward (positive values) distance from the central meridian and y the northward (positive values) distance from the equator (both measured in millimeters), with ϕ being the latitude and λ the longitude relative to the central meridian (both measured in radians).

Initially we started with polynomials of the form:

$$x, y = M \sum_i \sum_j c_{ij} \cdot \phi^i \cdot \lambda^j, \qquad (1)$$

with $i + j \leqslant 9$ and M being the numerical value of the map scale. Because the type of Equation (1) is linear in the coefficients c_{ij}, the latter can be determined by a multiple linear regression analysis. Because the independent variables (i.e. the $\phi^i\lambda^j$ terms) in Equation (1) are highly redundant, we used a stepwise regression approach to select only those terms being significant at the $p < 0.001$ level. This resulted in the following equations for the forward transformation:

$$\frac{y}{M} = a_0\phi + a_1\phi\lambda^2 + a_2\phi^3\lambda^2 + a_3\phi^5\lambda^2 \qquad (2)$$

and

$$\frac{x}{M} = b_0\lambda + b_1\phi^2\lambda + b_2\phi^4\lambda + b_3\phi^6\lambda + b_4\phi^2\lambda^3 + b_5\phi^4\lambda^3. \qquad (3)$$

The coefficients a_i and b_i in Equations (2) and (3) were estimated by a χ^2-fit of the data to these equations (e.g. Bevington and Robinson, 1992, p. 102). This approach considers the uncertainties of the measured cartesian coordinates (see previous). The coefficients and their errors are listed in Table 1. In order to evaluate the accuracy of the forward transformation, we applied the laws of error propagation (e.g. Bevington and Robinson, 1992, p. 41–48) to Equations (2) and (3) by assuming that the errors of the dependent variables are only the result of errors of the coefficients, that is assuming that the spherical coordinates are free of errors. Figure 1 shows the resulting uncertainties of the cartesian coordinates arising from the transformation. It can be seen from this figure that the maximum errors occur near the corners of the maps. The magnitude of the uncertainties differs between the x and y coordinate, being smaller for the latter. Although the errors in both directions are smaller than 0.2 mm for most parts of the maps, they may be larger than 1.0 mm near the map corners. However, the errors are sufficiently small for the purpose of setting up a paleoclimatic database, because most of the relevant (sedimentological) information is available from low and mid-latitudes sites.

INVERSE TRANSFORMATION

The goal of the inverse transformation is to determine the spherical coordinates for a given pair of cartesian coordinates. Because the spherical coordinates occur as nonlinear components in Equations (2) and (3), the inverse transformation has no unique solution. Hence, an iterative algorithm has to be applied, where Equations (2) and (3) are solved for different

Table 1. Polynomial coefficients used for Equations (2) and (3) and their errors

i	a_i	Δa_i	b_i	Δb_i
0	6366927995	1170111	5371771741	919898
1	23461742	824273	-1487638499	7691465
2	-94229585	1559661	372496080	9855206
3	23131445	746950	-94756618	3026031
4	-	-	-6180780	587882
5	-	-	1236742	541636

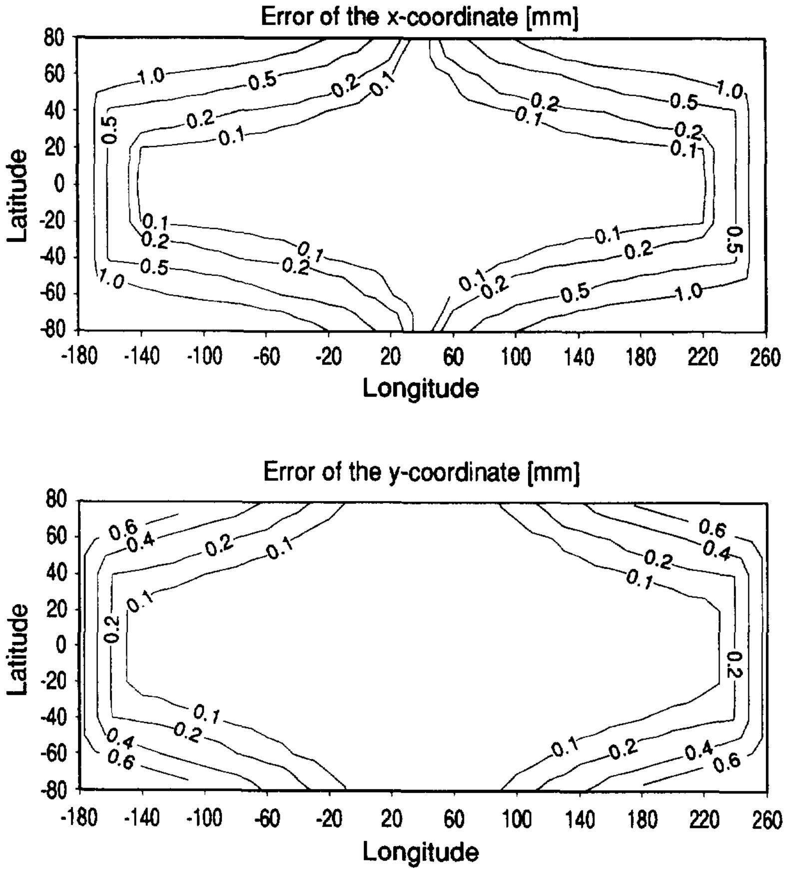

Figure 1. Lines of equal uncertainties of cartesian coordinates resulting from forward transformation. Numbers at contour lines denote errors in millimeters. It can be seen that errors are largest near corners of map and that uncertainties of y-coordinates generally are smaller than that of x-coordinates.

values of ϕ and λ until the difference between the calculated and the input values of x and y becomes smaller than a predefined value (usually in the order of 10^{-6} mm). For one independent variable this could be done by using the Newton–Raphson Method (e.g. Press and others, 1992, p. 362–367). However, the independent variables (latitude and longitude) occur in both equations of the forward transformation. Therefore, the numerical problem is that of determining the roots of a set of simultaneous nonlinear equations. This can be achieved using a modified Newton–Raphson algorithm as described in Press and others (1992, p. 383–389).

The selection of the equations for the forward transformation must be guided by the numerical stability of the inverse transformation. That is, the difference between the forward and inverse transformation must be close to zero for the entire range of possible input values, that is the algorithm must be globally convergent. Equations (2) and (3) are a compromise between numerical stability on the one hand and minimal lack of fit between equations and data on the other. To increase the numerical stability further, the inverse transformation is always carried out in the first map quadrant, that is for $x \geqslant 0$ mm and $y \geqslant 0$ mm. Making use of the symmetry of the map projection, the calculated spherical coordinates are mirrored into the correct quadrant. Initial values for the algorithm are determined by linear interpolation between the extreme values that occur in the first quadrant.

COMPUTER PROGRAM

The transformation program is written in Turbo Pascal 6.0 but it also should work with older versions. The executable program should run on any DOS compatible computer. The source code can be retrieved via the Internet from the IAMG anonymous ftp server, iamg.org.

Digitizing a map from the atlas involves several steps: (1) the forward transformation must be run for two or more points to calibrate the digitizing tablet; (2) the map then is digitized and the digitized information is saved in an ASCII file; and (3) the ASCII file is transformed into longitude–latitude coordinates using the inverse option of the program. As an example the digitized paleoshorelines for the Late Cretaceous are shown in Figure 2.

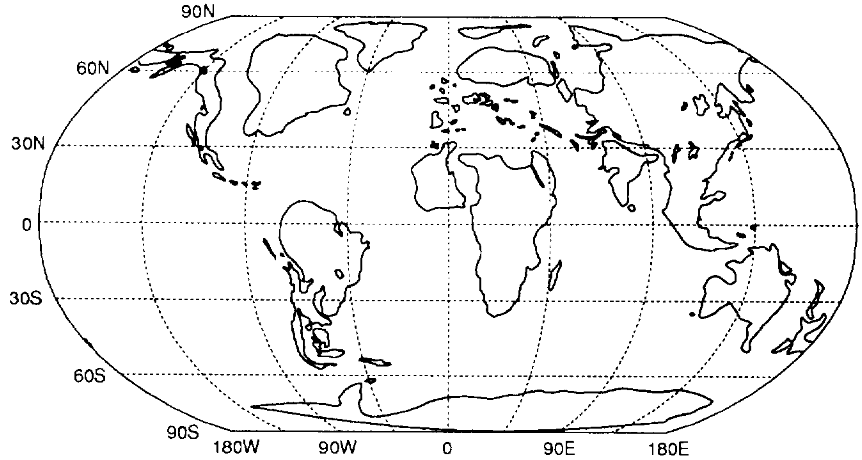

Figure 2. Late Cretaceous paleoshorelines (sealevel highstand) shown for present-day plate tectonic configuration. Data were digitized from Late Cretaceous map of Ronov, Khain, and Balukhovsky (1989). Subsequently inverse transformation was performed to convert cartesian coordinates into spherical coordinates that finally were plotted using Robinson projection.

The program reads cartesian coordinates (given in millimeters) from a space-delimited ASCII file with the following format:

$$\begin{array}{cc} x_1 & y_1 \\ x_2 & y_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ x_n & y_n \\ \multicolumn{2}{c}{-9999} \end{array}$$

The flag -9999 is used to separate different line segments in one data file. It should be noted that this flag is required if the data file contains only one line segment. The maximum number of digitized points per line segment is 8000.

Data are output using one of two formats. The first is a list of longitude–latitude values arranged in two space-delimited columns and the second is the FORTRAN (10F8.2) format used by the PC-ATLAS program for plate tectonic reconstructions (Cambridge Paleomap Services, 1990). This program is used widely for plate tectonic reconstructions on DOS-based computers.

Because of the longitude range used for the maps, some areas of the world appear twice. This leads to an ambiguity in the transformation procedure. The program handles this problem correctly for the inverse transformation. However for the forward transformation, only those points that lie between 180°W (on the left-hand margin of the map) and 180°E (on the right-hand side) can be calculated. For example, consider a point at 60°N, 160°W. This point appears twice on the maps, once on the left side with the cartesian coordinates $x_1 = -290.8$ mm, $y_1 = 181.2$ mm and once on the right side with $x_2 = 234.0$ mm, $y_2 = 166.0$ mm. However, inverse transformation yields the same longitude–latitude coordinates for both sets of input cartesian coordinates. Forward transformation of the longitude–latitude data will only determine the point x_1, y_1. However, using longitude = 200°E (equivalent to 160 W) yields the point x_2, y_2. This is a useful option for calibrating a digitizing tablet, because it allows the entire map to be used for calibration.

The program can be used interactively for the forward and inverse transformations of single points as well as for the inverse transformation of ASCII files containing digitized cartesian coordinates. The program can be used in batch mode for inverse transformations. Batch mode may be most appropriate for large or multiple data files. The syntax for batch mode is:

PALEOMAP Filename [-Fasc]

where [ ] denotes an optional parameter:

Filename—Complete path of the ASCII file containing the cartesian coordinates. The output file is named as described next.

F—Output file type: ASC = ASCII file, ATL = ATLAS file; default is ASC.

The name of the output file is the same as that of the input file but the filename extension is changed to '.DAT'. If the '.DAT' extension was used for the input file, then the output filename extension will be '.NEW'. A short help screen can be displayed by typing 'PALEOMAP?' on the command line.

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